

```

> restart;
> with(LinearAlgebra):
#xi1:=-1;
#eta1:=-1;
#xi2:=1;
#eta2:=-1;
#xi3:=1;
#eta3:=1;
#xi4:=-1;
#eta4:=1;
#a:=1.25;
#b:=1.00;
> B:=Matrix(3,8,[(-(1-eta1)/a),0,(1-eta2)/a,0,(1+eta3)/a,0,(-
(1+eta4)/a),0,
0,(-(1-xi1)/b),0,(-(1+xi2)/b),0,(1+xi3)/b,0,(1-xi4)/b,
(-(1-xi1)/b),(-(1-eta1)/a),(-(1+xi2)/b),(1-eta2)/a,
(1+xi3)/b,(1+eta3)/a,(1-xi4)/b,(-(1+eta4)/a)]);

```

$$B := \begin{bmatrix} -\frac{1-\eta_1}{a} & 0 & \frac{1-\eta_2}{a} & 0 & \frac{1+\eta_3}{a} & 0 & -\frac{1+\eta_4}{a} & 0 \\ 0 & -\frac{1-\xi_1}{b} & 0 & -\frac{1+\xi_2}{b} & 0 & \frac{1+\xi_3}{b} & 0 & \frac{1-\xi_4}{b} \\ -\frac{1-\xi_1}{b} & -\frac{1-\eta_1}{a} & -\frac{1+\xi_2}{b} & \frac{1-\eta_2}{a} & \frac{1+\xi_3}{b} & \frac{1+\eta_3}{a} & \frac{1-\xi_4}{b} & -\frac{1+\eta_4}{a} \end{bmatrix}$$

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B_i:=Matrix(2,3,[((xi1*(1+eta*eta1)/a)),0,((eta1*(1+xi*xi1)/b)
)),0,((eta1*(1+xi*xi1)/b)),((xi1*(1+eta*eta1)/a))]);

```

$$B_i := \begin{bmatrix} \frac{\xi_1(1+\eta\eta_1)}{a} & 0 & \frac{\eta_1(1+\xi\xi_1)}{b} \\ 0 & \frac{\eta_1(1+\xi\xi_1)}{b} & \frac{\xi_1(1+\eta\eta_1)}{a} \end{bmatrix}$$

```

> D_M:=Matrix(3,3,[1,v,0,v,1,0,0,0,((1-v)/2)]);

```

$$D_M := \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix}$$

```

> B_T := Transpose(B_i);

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$$B_T := \begin{bmatrix} \frac{\xi_1(1+\eta\eta_1)}{a} & 0 \\ 0 & \frac{\eta_1(1+\xi\xi_1)}{b} \\ \frac{\eta_1(1+\xi\xi_1)}{b} & \frac{\xi_1(1+\eta\eta_1)}{a} \end{bmatrix}$$

> **BB:=Multiply(B_i,D_M);**

$$BB := \begin{bmatrix} \frac{\xi_1 (1 + \eta \eta_1)}{a} & \frac{\xi_1 (1 + \eta \eta_1) v}{a} & \frac{\eta_1 (1 + \xi \xi_1) \left(\frac{1}{2} - \frac{v}{2}\right)}{b} \\ \frac{\eta_1 (1 + \xi \xi_1) v}{b} & \frac{\eta_1 (1 + \xi \xi_1)}{b} & \frac{\xi_1 (1 + \eta \eta_1) \left(\frac{1}{2} - \frac{v}{2}\right)}{a} \end{bmatrix}$$

> **Multiply(BB,B_T);**

$$\begin{bmatrix} \frac{\xi_1^2 (1 + \eta \eta_1)^2}{a^2} + \frac{\eta_1^2 (1 + \xi \xi_1)^2 \left(\frac{1}{2} - \frac{v}{2}\right)}{b^2}, \\ \frac{\xi_1 (1 + \eta \eta_1) v \eta_1 (1 + \xi \xi_1)}{a b} + \frac{\eta_1 (1 + \xi \xi_1) \left(\frac{1}{2} - \frac{v}{2}\right) \xi_1 (1 + \eta \eta_1)}{b a} \\ \left[\frac{\xi_1 (1 + \eta \eta_1) v \eta_1 (1 + \xi \xi_1)}{a b} + \frac{\eta_1 (1 + \xi \xi_1) \left(\frac{1}{2} - \frac{v}{2}\right) \xi_1 (1 + \eta \eta_1)}{b a}, \right. \\ \left. \frac{\eta_1^2 (1 + \xi \xi_1)^2}{b^2} + \frac{\xi_1^2 (1 + \eta \eta_1)^2 \left(\frac{1}{2} - \frac{v}{2}\right)}{a^2} \right] \end{bmatrix}$$

> **v:=0.2;**

Multiply(BB,B_T);

$$v := 0.2$$

$$\begin{bmatrix} \frac{\xi_1^2 (1 + \eta \eta_1)^2}{a^2} + \frac{0.4000000000 \eta_1^2 (1 + \xi \xi_1)^2}{b^2}, \\ \frac{0.6000000000 \xi_1 (1 + \eta \eta_1) \eta_1 (1 + \xi \xi_1)}{a b} \\ \left[\frac{0.6000000000 \xi_1 (1 + \eta \eta_1) \eta_1 (1 + \xi \xi_1)}{a b}, \right. \\ \left. \frac{\eta_1^2 (1 + \xi \xi_1)^2}{b^2} + \frac{0.4000000000 \xi_1^2 (1 + \eta \eta_1)^2}{a^2} \right] \end{bmatrix}$$

> **xi1:=-1;**

eta1:=-1;

#xi2:=1;

#eta2:=-1;

#xi3:=1;

#eta3:=1;

#xi4:=-1;

#eta4:=1;

```

a:=1.25;
b:=1.00;
#t:=0.1;
#v:=0.2;
#E:=30000;

```

```
ξ1 := -1
```

```
η1 := -1
```

```
a := 1.25
```

```
b := 1.00
```

```
> G:=Multiply(BB,B_T);
```

$$G := \begin{bmatrix} 0.6400000000 (1-\eta)^2 + 0.4000000000 (1-\xi)^2, & 0.4800000000 (1-\eta)(1-\xi) \\ 0.4800000000 (1-\eta)(1-\xi), & 1.0000000000 (1-\xi)^2 + 0.2560000000 (1-\eta)^2 \end{bmatrix}$$

```
> Int(Int(N,y=-1..1),x=-1..1);
```

$$\int_{-1}^1 \int_{-1}^1 N \, dy \, dx$$

```
> G[1,1];
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$$0.6400000000 (1-\eta)^2 + 0.4000000000 (1-\xi)^2$$

```
> A:=(1-y)^2/a^2;
```

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B:=+.4000000000*(1-x)^2/b^2;
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$$A := 0.6400000000 (1-y)^2$$

$$B := 0.4000000000 (1-x)^2$$

```
> Int(Int(A+B,y=-1..1),x=-1..1);
```

$$\int_{-1}^1 \int_{-1}^1 0.6400000000(1-y)^2 + 0.4000000000(1-x)^2 \, dy \, dx$$

```
> k_11:=int(int(A+B,y=-1..1),x=-1..1);
```

$$k_{11} := 5.546666667$$

```
> k:=((a*b*t)/16)*(E/(1-v^2));
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$$k := 0.08138020831t E$$

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> k_11*k;
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$$0.4513888888t E$$

```
> a:=1.25;
```

```
b:=1.00;
```

```
t:=0.1;
```

```
v:=0.2;
```

```
E:=30000;
```

```
k_11:=int(int(A+B,y=-1..1),x=-1..1);
```

```
k:=((a*b*t)/16)*(E/(1-v^2));
```

```
k_11*k;
```

$a := 1.25$

$b := 1.00$

$t := 0.1$

$v := 0.2$

$E := 30000$

$k_{11} := 5.546666667$

$k := 244.1406250$

1354.166667

>